

Dark energy from modified $F(R)$ -scalar-Gauss-Bonnet gravity

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The modified $F(R)$ -scalar-Gauss-Bonnet gravity is proposed as dark energy model. The reconstruction program for such theory is developed. It is explicitly demonstrated that the known classical universe expansion history (deceleration epoch, transition to acceleration and effective quintessence, phantom or cosmological constant era) may naturally occur in such unified theory for some (reconstructed) classes of scalar potentials. Gauss-Bonnet assisted dark energy is also proposed. The possibility of cosmic acceleration is studied there.

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I. INTRODUCTION

It is quite possible that dark energy is some manifestation of the unknown gravitational physics. The search of the realistic gravitational alternative for dark energy (for a recent review, see [1]) continues. According to this approach, usual General Relativity which was valid at deceleration epoch should be modified by some gravitational terms which became relevant at current, accelerating universe when curvature is getting smaller. Among the most popular modified gravities which may successfully describe the cosmic speed-up is $F(R)$ gravity. Very simple versions of such theory like $1/R$ [2] and $1/R + R^2$ [3] may lead to the effective quintessence/phantom late-time universe. Moreover, positive/negative powers of curvature in the effective gravitational action may have stringy/M-theory origin [4]. Recently, big number of works was devoted to the study of late-time cosmology and solar system tests checks in modified $F(R)$ gravity [5, 6]. While it is not easy to satisfy all known solar system tests at once, it is possible to construct the explicit models [7, 8] which describe the realistic universe expansion history (radiation/matter dominance, transition to acceleration and accelerating era).

Another theory proposed as gravitational dark energy is scalar-Gauss-Bonnet gravity [9] which is closely related with low-energy string effective action. The late-time universe evolution and comparison with astrophysical data in such model was discussed in refs.[9, 10, 11, 12]. The possibility to extend such consideration to third order (curvature cubic) terms in low-energy string effective action exists [13]. Moreover, one can develop the reconstruction program for such theories (see [14], for a review). It has been demonstrated [15] that some scalar-Gauss-Bonnet gravities may be compatible with the known classical history of the universe expansion.

In the present paper, we propose the unified $F(R)$ -scalar-Gauss-Bonnet gravity as dark energy model. The reconstruction program for such model is explicitly developed. It is shown that it is easier to realize the known, classical universe history (deceleration, transition to acceleration and cosmic acceleration with effective w close to -1) in such a model than in $F(R)$ or scalar-Gauss-Bonnet gravity. Several different late-time regimes (effective quintessence, phantom or cosmological constant) are investigated. Moreover, it is indicated that it is possible to pass Solar System tests in such a unified model. It is shown also that Gauss-Bonnet term may play an important role for other class of gravitational models where it couples with scalar kinetic term. The possibility of cosmic acceleration in such model is also demonstrated.

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II. COSMIC ACCELERATION IN $F(R)$ -SCALAR-GAUSS-BONNET GRAVITY

In this section, let us introduce $F(R)$ -scalar-Gauss-Bonnet gravity whose action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + F(R) - \frac{\eta}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \xi(\phi)G + L_{\text{matter}} \right]. \quad (1)$$

Here $F(R)$ is a function of the scalar curvature R , L_{matter} is matter Lagrangian density, and G is GB invariant: $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$.

In (1), with $F(R) = 0$ $\eta = 1$ corresponds to string-inspired scalar-Gauss-Bonnet gravity which has been proposed as dark energy model [9] (it may be applied also for resolution of the initial singularity in early universe [16]) and $\eta = 0$ to $f(G)$ gravity [17]. Note that using trick of ref.[3] one can delete $F(R)$ term, at the price of the introduction of second scalar. However, in such formulation the coupling (including derivatives) of new scalar with GB sector appears.

In the following, the metric is assumed to be in the spatially-flat FRW form:

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^3 (dx^i)^2. \quad (2)$$

The FRW field equations look as

$$0 = -\frac{R}{2\kappa^2} - F(R) + 6 \left(H^2 + \dot{H} \right) \left(\frac{1}{2\kappa^2} + F'(R) \right) - 36 \left(4H^2 \dot{H} + H \ddot{H} \right) F''(R) \\ + \frac{1}{2} \dot{\phi}^2 + V(\phi) + 24H^3 \frac{d\xi(\phi(t))}{dt} + \rho_{\text{matter}}, \quad (3)$$

$$0 = \frac{R}{2\kappa^2} + F(R) - 2 \left(\dot{H} + 3H^2 \right) \left(\frac{1}{2\kappa^2} + F'(R) \right) \\ + 48 \left(4H^2 \dot{H} + \dot{H}^2 + 2H \ddot{H} \right) F''(R) + 72 \left(4H \dot{H} + \ddot{H} \right) F'''(R) + \frac{1}{2} \dot{\phi}^2 - V(\phi) \\ - 8H^2 \frac{d^2 \xi(\phi(t))}{dt^2} - 16H \dot{H} \frac{d\xi(\phi(t))}{dt} - 16H^3 \frac{d\xi(\phi(t))}{dt} + p_{\text{matter}}. \quad (4)$$

and the scalar field equation is

$$0 = \eta \left(\ddot{\phi} + 3H\dot{\phi} \right) + V'(\phi) + \xi'(\phi)G. \quad (5)$$

Here $R = 12H^2 + 6\dot{H}$ and $G = 24 \left(\dot{H}H^2 + H^4 \right)$

We now consider the perfect fluids with constant equation of state (EoS) parameters $w_i \equiv p_i/\rho_i$ as the matter. The energy density is $\rho_i = \rho_{i0} a^{-3(1+w_i)}$ with a constant ρ_{i0} . Let us parametrize the model with two proper functions $f(\phi)$ and $g(t)$ as follows (compare with [15] for $F = 0$ case),

$$V(\phi) = \frac{\hat{R}}{2\kappa^2} + F(\hat{R}) - 6 \left(g'(f(\phi))^2 + g''(f(\phi)) \right) \left(\frac{1}{2\kappa^2} + F'(\hat{R}) \right) \\ + 36 \left(4g'(f(\phi))^2 g''(f(\phi)) + g'(f(\phi)) g'''(f(\phi)) \right) F''(\hat{R}) - 3g'(f(\phi)) e^{g(f(\phi))} U(\phi), \\ \xi(\phi) = \frac{1}{8} \int^\phi d\phi_1 \frac{f'(\phi_1) e^{g(f(\phi_1))} U(\phi_1)}{g'(f(\phi_1))^2}, \\ U(\phi) \equiv \int^\phi d\phi_1 f'(\phi_1) e^{-g(f(\phi_1))} \left(4g''(f(\phi_1)) \left(\frac{1}{2\kappa^2} + F'(\hat{R}) \right) \right. \\ + 12 \left(4g'(f(\phi_1))^2 g''(f(\phi_1)) + 4g''(f(\phi_1))^2 + 5g'(f(\phi_1)) g'''(f(\phi_1)) \right) F''(\hat{R}) \\ + 72 \left(4g'(f(\phi_1)) g''(f(\phi_1)) + g'''(f(\phi_1)) \right) F'''(\hat{R}) \\ \left. + \frac{\eta}{f'(\phi_1)^2} + \sum_i (1 + w_i) \rho_{i0} a_0^{-3(1+w_i)} e^{-3(1+w_i)g(f(\phi_1))} \right), \\ \hat{R} \equiv 12g'(f(\phi))^2 + 6g''(f(\phi)). \quad (6)$$

Here, the equations have the following solution:

$$a = a_0 e^{g(t)} \quad (H = g'(t)) \quad , \quad \phi = f^{-1}(t) \quad . \quad (7)$$

This may be considered as reconstruction of above modified gravity from known universe history expansion.

In case $\eta = 0$ in (1), we can freely redefine the scalar field as $\phi \rightarrow \phi' = \phi'(\phi)$. If ϕ depends on the time t as $\phi = \phi(t)$, one can redefine the scalar field properly and identify the scalar field as t , $\phi = t$, that is $f(\phi) = \phi$. Then Eq.(6) can be simplified as (for the case $F = 0$ compare with [14])

$$\begin{aligned} V(\phi) &= \frac{\hat{R}}{2\kappa^2} + F(\hat{R}) - 6 \left(g'(\phi)^2 + g''(\phi) \right) \left(\frac{1}{2\kappa^2} + F'(\hat{R}) \right) \\ &\quad + 36 \left(4g'(\phi)^2 g''(\phi) + g'(\phi) g'''(\phi) \right) F''(\hat{R}) - 3g'(\phi) e^{g(\phi)} U(\phi) \quad , \\ \xi(\phi) &= \frac{1}{8} \int^\phi d\phi_1 \frac{e^{g(\phi_1)} U(\phi_1)}{g'(\phi_1)^2} \quad , \\ U(\phi) &\equiv \int^\phi d\phi_1 e^{-g(\phi_1)} \left(4g''(\phi_1) \left(\frac{1}{2\kappa^2} + F'(\hat{R}) \right) \right. \\ &\quad + 12 \left(4g'(\phi_1)^2 g''(\phi_1) + 4g''(\phi_1)^2 + 5g'(\phi_1) g'''(\phi_1) \right) F''(\hat{R}) \\ &\quad + 72 \left(4g'(\phi_1) g''(\phi_1) + g'''(\phi_1) \right) F'''(\hat{R}) \\ &\quad \left. + \sum_i (1 + w_i) \rho_{i0} a_0^{-3(1+w_i)} e^{-3(1+w_i)g(\phi_1)} \right) \quad , \\ \hat{R} &\equiv 12g'(\phi)^2 + 6g''(\phi) \quad . \end{aligned} \quad (8)$$

As some example of $F(R)$, we consider [2]

$$F(R) = -\frac{\mu^4}{R} \quad . \quad (9)$$

The original model has two solutions corresponding to deSitter universe, where R is constant, and asymptotic universe with effective $w = -2/3$. In the model (6), one can realize any time development of the universe. For example, by choosing (9) and

$$g(t) = H_0 t + H_1 \ln \left(\frac{t}{t_0} \right) \quad , \quad (10)$$

and $f(\phi)$ to be properly defined, we obtain

$$H(t) = H_0 + \frac{H_1}{t} \quad . \quad (11)$$

When t is small, H (11) behaves as that in universe with perfect fluid with $w = -1 + 2/3H_1$ and when t is large, H behaves as in deSitter space, where H is a constant. Then if we choose $H_1 = 2/3$, we find that before the acceleration epoch, the universe behaves as matter dominated one with $w = 0$. After that, it enters to acceleration phase. In the original model [2], it was difficult to realize the matter dominated phase. It is easy to see that matter dominance phase (with subsequent acceleration) can be easily realized by adding the Gauss-Bonnet term (see Appendix for explicit form of scalar potentials).

As another example, we consider a model with (9) but

$$g(t) = \tilde{H}_0 \ln \frac{t}{t_0} - \tilde{H}_1 \ln \left(\frac{t_0 - t}{t_0} \right) \quad , \quad (12)$$

which gives

$$H(t) = \frac{\tilde{H}_0}{t} + \frac{\tilde{H}_1}{t_0 - t} \quad . \quad (13)$$

Here \tilde{H}_0 , \tilde{H}_1 , and t_0 are positive constants. When t is small, H (13) behaves in a way corresponding to the perfect fluid with $w = -1 + 2/3\tilde{H}_0$. Then if we choose $\tilde{H}_0 = 2/3$, we find the matter dominated universe. On the other

hand, when $t \sim t_0$ is large, H behaves as that in the phantom universe with $w = -1 - 2/3\tilde{H}_1 < -1$ and there will appear a big rip singularity at $t = t_0$. The three-year WMAP data are analyzed in Ref. [18], which show that the combined analysis of WMAP with the supernova Legacy survey (SNLS) constrains the dark energy equation of state w_{DE} pushing it clearly towards the cosmological constant value. The marginalized best fit values of the equation of state parameter at 68% confidence level are given by $-1.14 \leq w_{DE} \leq -0.93$, which corresponds to $\tilde{H}_1 > 10.7$ as \tilde{H}_1 is positive. In case one takes as a prior that the universe is flat, the combined data gives $-1.06 \leq w_{DE} \leq -0.90$, which corresponds to $\tilde{H}_1 > 25.0$. Therefore the possibility that $w_{DE} < -1$ has not been excluded. As clear from (6) or (8), the expressions of $V(\phi)$ and $\xi_1(\phi)$ depend on the explicit form of $F(R)$, say μ in (9), and the time-development of the universe $g(t)$, now \tilde{H}_0 and \tilde{H}_1 , (and $f(\phi)$ in case of (6)). Then the form of $F(R)$ is irrelevant to the WMAP data but only $g(t)$ is relevant as long as we use the expressions in (6) or (8).

One may also consider a model

$$g(t) = \hat{H}_0 \ln \frac{t}{t_0} + (\hat{H}_1 - \hat{H}_0) \ln \left(\frac{t_0 + t}{t_0} \right), \quad (14)$$

with constants \hat{H}_0 , \hat{H}_1 , and t_0 . Then we obtain

$$H(t) = \frac{\hat{H}_0}{t} + \frac{\hat{H}_1 - \hat{H}_0}{t_0 + t}. \quad (15)$$

When t is small, H again behaves as $H \sim \hat{H}_0/t$ corresponding to the universe filled with perfect fluid $w = -1 + 2/3\hat{H}_0$. On the other hand, when $t \sim t_0$ is large, H behaves as $H \sim \hat{H}_1/t$ corresponding to the fluid with $w = -1 + 2/3\hat{H}_1$. Therefore with the choice $\hat{H}_0 = 2/3$, we find the matter dominated universe in the early universe and with the choice $\hat{H}_1 > 1$, we obtain acceleratedly expanding universe, where $w < -1/3$. The constraint from only the WMAP data indicates $\hat{H}_1 > 21.4$ and that from the combined data indicates $\hat{H}_1 > 15.0$.

One more example is Λ CDM-type cosmology:

$$g(t) = \frac{2}{3(1+w)} \ln \left(\alpha \sinh \left(\frac{3(1+w)}{2l} (t - t_0) \right) \right), \quad \alpha^2 \equiv \frac{1}{3} \kappa^2 l^2 \rho_0 a_0^{-3(1+w)}. \quad (16)$$

Here l is the length scale given by cosmological constant $l \sim (10^{-33} \text{ eV})^{-1}$ and t_0 is a constant. The time-development of the universe given by $g(t)$ (16) can be realized in the usual Einstein gravity with a cosmological constant Λ and cold dark matter (CDM), which could be regarded as dust. Then in the present formulation, by using $V(\phi)$ and $\xi_1(\phi)$ in (6) or (8), Λ CDM-type cosmology can be realized *without* introducing CDM as a matter.

One may also consider more general choice of $F(R)$ as in [3]

$$F(R) = -\frac{\alpha}{R} + \beta R^2, \quad (17)$$

or

$$F(R) = -\frac{\tilde{\alpha}}{R^n} + \tilde{\beta} R^m. \quad (18)$$

Even in this case, we can realize any time development of the universe. For example, if we choose $g(t)$ as in (10), H (11) behaves as that in universe with perfect fluid with $w = -1 + 2/3H_1$ when t is small (especially by choosing $H_1 = 2/3$, the matter dominated phase occurs) and H behaves as in deSitter space when t is large. On the other hand, if we choose $g(t)$ as in (12), we obtain a model showing the transition from the matter dominated phase to the phantom phase. Moreover, with the choice of (15), the transition from the matter dominated phase to the quintessence phase could be realized.

Let us now consider the string-inspired model[9], where

$$V = V_0 e^{-\frac{2\phi}{\phi_0}}, \quad \xi(\phi) = \xi_0 e^{\frac{2\phi}{\phi_0}}, \quad (19)$$

with constant parameters V_0 , ξ_0 , and ϕ_0 (with $F(R)$ given by (17)). Different from the model (6), it is not straightforward to solve the FRW equations (3), (4) and the scalar field equation in this model and to find the behavior of the universe. In case $|\frac{\alpha}{R}|$, $|\beta R^2| \ll |\frac{R}{\kappa^2}|$, the cosmology [9] could be reproduced, that is, if we choose,

$$\begin{aligned} V_0 t_1^2 &= -\frac{1}{\kappa^2 (1+h_0)} \left\{ 3h_0^2 (1-h_0) + \frac{\phi_0^2 \kappa^2 (1-5h_0)}{2} \right\}, \\ \frac{48\xi_0 h_0^2}{t_1^2} &= -\frac{6}{\kappa^2 (1+h_0)} \left(h_0 - \frac{\phi_0^2 \kappa^2}{2} \right). \end{aligned} \quad (20)$$

we obtain a solution

$$H = \frac{h_0}{t}, \quad \phi = \phi_0 \ln \frac{t}{t_1}, \quad (21)$$

when $h_0 > 0$ or

$$H = -\frac{h_0}{t_s - t}, \quad \phi = \phi_0 \ln \frac{t_s - t}{t_1}, \quad (22)$$

when $h_0 < 0$ and a constant t_1 . We should also note that there could also be deSitter solution: $H = H_0$, $\phi = p_0$. Then the FRW equation (3) and the scalar field equation (5) have the following simple forms:

$$0 = -\frac{3}{\kappa^2} H_0^2 + \frac{\alpha}{8H_0^2} + V_0 e^{-2p_0/\phi_0}, \quad 0 = V_0 e^{-2p_0/\phi_0} - \xi_0 e^{2p_0/\phi_0}. \quad (23)$$

When V_0 and ξ_0 are positive, by combining two equations in (23), we obtain

$$0 = -\left(\frac{3}{\kappa^2} - 2\sqrt{6V_0\xi_0}\right) H_0^2 + \frac{\alpha}{8H_0^2}. \quad (24)$$

If α is positive and $\frac{3}{\kappa^2} > 2\sqrt{6V_0\xi_0}$, there can be a solution $H_0^4 = \frac{\alpha}{8(\frac{3}{\kappa^2} - 2\sqrt{6V_0\xi_0})}$, which gives $e^{2p_0/\phi_0} = \sqrt{(\frac{3}{\kappa^2} - 2\sqrt{6V_0\xi_0}) \frac{V_0}{3\alpha\xi_0}}$. Therefore, in general, there could occur deSitter solution as late-time universe.

Hence, we demonstrated that in unified $F(R)$ -scalar-GB gravity which may be considered as string-inspired theory one may have the number of dark energy scenarios of different type (effective quintessence, effective cosmological constant or effective phantom). The corresponding classes of scalar potentials may be easily constructed (see an explicit example of such construction in the Appendix). It is interesting that in such unified model it is easier to realize the known classical universe history (radiation/matter dominance, deceleration-acceleration transition, cosmic acceleration) than in pure $F(R)$ gravity [7, 8] or in pure scalar-GB gravity [14, 15]. There is no problem to take into account usual matter in such consideration. In this case it is even easier to reconstruct the deceleration (radiation/matter dominance) phase before the acceleration epoch. However, the explicit form of corresponding potentials is quite complicated.

III. GAUSS-BONNET GRAVITY ASSISTED DARK ENERGY

In this section we study modified gravity model motivated by gravity assisted dark energy [19] where scalar kinetic term couples with the function of Gauss-Bonnet invariant. It is different from the model of previous section, but GB term plays again the important role here. The starting action is:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{\kappa^2} R - f(G) L_d \right], \quad (25)$$

where $L_d = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. Taking the same FRW metric and assuming ϕ only depends on time coordinate t , $\phi = \phi(t)$, we find the solution of scalar field equation:

$$\dot{\phi} = f(G)^{-1} a^{-3} q. \quad (26)$$

Here q is a constant.

The FRW field equation is found to be:

$$\frac{6}{\kappa^2} H^2 - \frac{q^2}{a^6 f(G)} \left[\frac{1}{2} + 12H^2(\dot{H} + 7H^2) \frac{f'(G)}{f(G)} + 24^2 \left(\frac{f'(G)}{f(G)} \right)^2 H^4 (H\ddot{H} + 4\dot{H}H^2 + 2\dot{H}^2) - 12H^3 \frac{\frac{d}{dt}f'(G)}{f(G)} \right] = 0. \quad (27)$$

Here $f' \equiv df(G)/dG$. The following choice of $f(G)$ is convenient: $f(G) = \left(\frac{G}{\mu^4}\right)^\alpha$. Here α (non-integer) and μ^4 are constants. Then in order that $f(G)$ is real, we only consider the case that G has a definite sign and when G is positive (negative), $\mu^4 > 0$ ($\mu^4 < 0$). One can see that for $\alpha = -1$, FRW equation takes the very simple form:

$$\frac{H^2}{\kappa^2} + \frac{288q^2 H^4}{\mu^4 a^6} = 0. \quad (28)$$

When $\mu^4 > 0$, the equation has only trivial solution $H = 0$. When $\mu^4 < 0$, the following solution exists:

$$a(t) = 2^{2/3} \left(-\frac{2q^2}{\mu^4} \right)^{1/6} t^{1/3}. \quad (29)$$

which gives $H = 1/3t$ and $G = -16/27t^4$.

Now for general α , by using a constant x , we assume

$$a = a_0 t^x, \quad \left(H = \frac{x}{t} \right). \quad (30)$$

Then it follows $x = \frac{2\alpha+1}{3}$, $a_0^6 = 48q^2\kappa^2 \left\{ \frac{81\mu^4}{(2\alpha+1)(\alpha-1)} \right\}^\alpha \frac{(\alpha^2-3\alpha-1)}{(\alpha-1)}$. Since $G = \frac{16(2\alpha+1)(\alpha-1)}{27t^4}$, we find $\mu^4 > 0$ when $\alpha > 1$ or $\alpha < -1/2$ and $\mu^4 < 0$ when $-1/2 < \alpha < 1$. Since $\ddot{a} = \frac{2}{9}a_0(2\alpha+1)(\alpha-1)t^{\frac{2\alpha-5}{3}}$ the universe accelerates ($\ddot{a} > 0$) if $\alpha > 1$ or $\alpha < -1/2$. We should also note that $81\mu^4/(2\alpha+1)(\alpha-1)$ is positive by assumption. Since $a_0^6 > 0$, one gets $(3 - \sqrt{13})/2 < \alpha < 1$ or $\alpha > (3 + \sqrt{13})/2$. Hence, if $(3 - \sqrt{13})/2 < \alpha < 1$, the expansion of the universe is decelerating but if $\alpha > (3 + \sqrt{13})/2$, the expansion is accelerating.

The effective equation of state parameter in our case is $w = \frac{1-2\alpha}{1+2\alpha}$. If $\alpha < -\frac{1}{2}$ the effective phantom era occurs, while if $\alpha > 1$ the effective quintessence era emerges.

Let us add the perfect fluid $p = (\gamma - 1)\rho$, where $\gamma \in (0, 2)$, to our system. The FRW equation may be presented in the following form:

$$\frac{H^2}{\kappa^2} = \rho + \rho_G, \quad (31)$$

where

$$\rho_G = \frac{48q^2\mu^{4\alpha}}{a^6(\dot{H}H^2 + H^4)^{\alpha+2}} \left[\alpha(\alpha+1)\ddot{H}H^5 + 2(2\alpha^2 + 6\alpha + 1)\dot{H}H^6 + 2(\alpha+1) \left(\alpha + \frac{1}{2} \right) \dot{H}^2H^4 + (1 + 7\alpha)H^8 \right]. \quad (32)$$

The energy density ρ satisfies the equation $\dot{\rho} = -3\gamma H\rho$. From (31), it follows $\rho = H^2/\kappa^2 - \rho_G$. Substituting it into the conservation law, one gets: $2\frac{\dot{H}H}{\kappa^2} - \dot{\rho}_G + 3\gamma\frac{H^3}{\kappa^2} - 3\gamma H\rho_G = 0$. By assuming (30), we find $a = a_0 t^{\frac{1+2\alpha}{3}}$ as in the case without the perfect fluid and $\rho = \rho_0 t^{-\gamma(1+2\alpha)}$. If $\rho \propto \rho_G$, we find $\gamma(1+2\alpha) = 2$. For the case that matter is dust with $\gamma = 1$, if $\alpha > \frac{1}{2}$, the matter could dominate in the early universe ($t \rightarrow 0$) and ρ_G dominates in the late universe $t \rightarrow +\infty$. Hence, if $\alpha > (3 + \sqrt{13})/2$, there could occur the transition from matter dominated phase to the accelerated expansion. It is interesting to note that de Sitter solution is impossible in this scenario with only ρ_G .

Thus, we demonstrated the possibility to realize the Gauss-Bonnet gravity assisted dark energy with late-time cosmic acceleration. The deceleration era may naturally emerge in such scenario before the acceleration with subsequent transition to acceleration.

IV. DISCUSSION

In summary, we studied late-time, dark energy era in $F(R)$ -scalar-Gauss-Bonnet gravity. The reconstruction method for such model is developed. It is shown that it may be reconstructed from the known universe expansion history so that for some class of scalar potentials the radiation/matter dominance is realized subsequently transiting to cosmic acceleration. Moreover, any type of cosmic speed-up: the effective quintessence, phantom or Λ -CDM era may occur after deceleration epoch. It is remarkable that it is easier to achieve the deceleration era transiting to dark era than in $F(R)$ or scalar-Gauss-Bonnet gravity.

Gauss-Bonnet gravity assisted dark energy is proposed. It is shown that cosmic acceleration may naturally occur in such theory as well. It is interesting that adding the scalar potential to kinetic term in such theory one can construct the Gauss-Bonnet induced model for the dynamical origin of the effective cosmological constant in close analogy with ref.[19].

Let us discuss now solar system tests for our model. It has been argued recently that $F(R)$ -gravity does not pass the solar system tests [20]. Recently, however, in [21], the conditions that even pure $F(R)$ -gravity (of special form) could satisfy the solar system and cosmological tests are derived. The conditions are:

$$\lim_{R \rightarrow \infty} F(R) = \text{const}, \quad \lim_{R \rightarrow 0} F(R) = 0. \quad (33)$$

An explicit example of such function (with specific values of parameters) is given by

$$F(R) = -\frac{m^2 c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}. \quad (34)$$

Hence, we can choose $F(R)$ as (34) and then apply the formulations (6) (for $\eta = 1$ case) and (8) (for $\eta = 0$ case) and therefore we can realize arbitrary (accelerating) cosmology by properly choosing $V(\phi)$ and $\xi(\phi)$. In this case, our theory passes solar system tests. For $F(G)$ -gravity, which corresponds to $\eta = 0$ case, it has been shown that the scalar field ϕ does not propagate and the only propagating mode is graviton, at least in the de Sitter background. The model proposed in this paper is the hybrid of $F(R)$ -gravity and the scalar-Gauss-Bonnet or $F(G)$ -gravity. Hence, the above arguments indicate that cosmological and solar system tests could be satisfied at once if we choose $F(R)$ (as above), $V(\phi)$, and $\xi(\phi)$, properly. Of course, to satisfy all cosmological tests some tuning of parameters of our theory may be necessary what should be checked explicitly for any particular model.

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APPENDIX A

In this Appendix for the simple cosmology case (9) and (10), we present the explicit form of $V(\phi)$ and ξ_1 .

For the $F(R)$ -scalar-Gauss-Bonnet model ($\eta = 1$ in (9)), we further choose $f(\phi) = f_0\phi$. Then $V(\phi)$ and ξ_1 (6) have the following form:

$$\begin{aligned} V(\phi) &= \frac{\hat{R}}{2\kappa^2} - \frac{\mu^4}{12 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 - \frac{6H_1}{f_0^2\phi^2}} - 6 \left(\left(H_0 + \frac{H_1}{f_0\phi}\right)^2 - \frac{H_1}{f_0^2\phi^2} \right) \left(\frac{1}{2\kappa^2} + \frac{\mu^4}{\left(12 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 - \frac{6H_1}{f_0^2\phi^2}\right)^2} \right) \\ &\quad + 72 \left(-4 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 \frac{H_1}{f_0^2\phi^2} + 2 \left(H_0 + \frac{H_1}{f_0\phi}\right) \frac{H_1}{f_0^3\phi^3} \right) \frac{\mu^4}{\left(12 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 - \frac{6H_1}{f_0^2\phi^2}\right)^3} \\ &\quad - 3 \left(H_0 + \frac{H_1}{f_0\phi}\right) e^{H_0 f_0 \phi} \left(\frac{f_0\phi}{t_0}\right)^{H_1} U(\phi), \\ \xi_1(\phi) &= \frac{1}{8} \int^\phi d\phi_1 \frac{f_0 e^{H_0 f_0 \phi} \left(\frac{f_0\phi}{t_0}\right)^{H_1} U(\phi_1)}{\left(H_0 + \frac{H_1}{f_0\phi}\right)^2}, \\ U(\phi) &\equiv \int^\phi d\phi_1 f_0 e^{-H_0 f_0 \phi_1} \left(\frac{f_0\phi_1}{t_0}\right)^{-H_1} \left(-4 \frac{H_1}{f_0^2\phi_1^2} \left(\frac{1}{2\kappa^2} + \frac{\mu^4}{\left(12 \left(H_0 + \frac{H_1}{f_0\phi_1}\right)^2 - \frac{6H_1}{f_0^2\phi_1^2}\right)^2} \right) \right. \\ &\quad \left. - 24 \left(-4 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 \frac{H_1}{f_0^2\phi^2} + 4 \frac{H_1^2}{f_0^4\phi_1^4} + 10 \left(H_0 + \frac{H_1}{f_0\phi_1}\right) \frac{H_1}{f_0^3\phi^3} \right) \frac{\mu^4}{\left(12 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 - \frac{6H_1}{f_0^2\phi^2}\right)^3} \right. \\ &\quad \left. + 432 \left(-4 \left(H_0 + \frac{H_1}{f_0\phi}\right) \frac{H_1}{f_0^2\phi^2} + \frac{H_1}{f_0^3\phi^3} \right) \frac{\mu^4}{\left(12 \left(H_0 + \frac{H_1}{f_0\phi}\right)^2 - \frac{6H_1}{f_0^2\phi^2}\right)^4} \right) \end{aligned}$$

$$+ \frac{\eta}{f_0^2} + \sum_i (1 + w_i) \rho_{i0} a_0^{-3(1+w_i)} e^{-3(1+w_i)H_0 f_0 \phi} \left(\frac{f_0 \phi}{t_0} \right)^{-3(1+w_i)H_1} \Bigg). \quad (\text{A1})$$

This defines the class of scalar potentials which lead to above cosmological solution. For $f(G)$ gravity with $\eta = 0$, the similar form of $V(\phi)$ and $\xi(\phi)$ may be derived.

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